Improving Stochastic Estimator Techniques for Disconnected Diagrams*

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Disconnected diagrams are expected to be sensitive to the inclusion of dynamical fermions. We present a feasibility study for the observation of such effects on the nucleonic matrix elements of the axial vector current, using SESAM full QCD vacuum configurations with Wilson fermions on $16^3 \times 32$ lattices, at $\beta = 5.6$. Starting from the standard methods developed by the Kentucky and Tsukuba groups, we investigate the improvement from various refinements thereof.

1. Introduction

The calculation of hadronic matrix elements containing flavor singlet quark bilinear operators $\mathcal{O} = \bar{q}\Gamma q$ is expected to reveal sea quark effects in full QCD simulations.

The standard method for such calculations uses the ratio of the three- and the two-point functions:

$$R(t) = \frac{\langle N(t)\sum_n \mathcal{O}(n)\bar{N}(0)\rangle}{\langle N(t)\bar{N}(0)\rangle} \xrightarrow{t\gg 1} const + Z_0^{-1}\langle N|\mathcal{O}|N\rangle\, t\,, \quad (1)$$

where Z_0 is the lattice renormalisation factor for $\mathcal{O}$. In the limit of large t $R(t)$ shows a constant slope, which determines the matrix element under investigation. In this work we focus on scalar and axial vector insertions ($\Gamma = 1$, $\Gamma = \gamma_k\gamma_5$) needed for the determination of the $\pi N\sigma$ term, which measures chiral symmetry breaking and for the axial coupling constant, g_A^1. In both cases connected and disconnected diagrams have to be dealt with. For the former suitable methods are available. However, the latter present a severe bottleneck in form of the computation of $\Delta = (\Gamma M^{-1})_{x\to x}$ which is of complexity $\mathcal{O}$(volume)[1].

2. Improving Scalar Insertions

To tackle the bottleneck several estimator techniques have been devised by different groups [2–4]. All these methods approximate Δ by scalar products $(\eta, \Gamma M^{-1}\eta)$ with appropriate source vectors η. In the so called wall source method a homogeneous 4-volume source is used[4], leading to unbiased observables only in the limit of an infinite ensemble of gauge configurations. In view of the cost of sampling this poses a non trivial problem for full QCD simulations.

This problem is alleviated by the stochastic estimator techniques (SET)[2,3].

It has been shown previously[6] that a complex Z_2 noise source is superior to Gaussian noise, in the case of the scalar insertion. In this contribution we present new methods to improve on the signal to noise ratio for scalar and axial vector insertions.

One possible strategy to reduce the noise is to restrict the summation over n in eq. 1 to the plateau region of the proton correlator, i.e. to the ground state regime of the proton operator. We refer to this approach as PSQL (plateau sampling of quark loops).

The efficiency of PSQL (with $1 \leq n_t \leq t-1$) is illustrated in figure 1, where we compare with standard SET. Note that the signal to noise ratio is improved by a factor $2-3$.

3. Axial insertions

The matrix element of the axial vector current between proton states (for zero momentum) can

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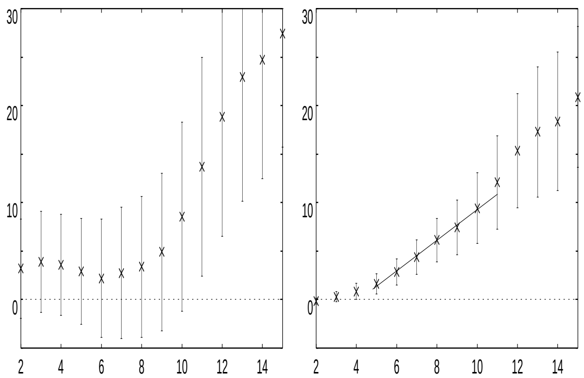

Figure 1. *$R(t)$ for the scalar insertion measured without PSQL (left) and with PSQL on* 200 *SESAM configurations,* 100 *Z2 noise estimates,* $\kappa = 0.157$.

be written as:

$$\langle P|\bar{q}_i\gamma_\mu\gamma_5 q_i|P\rangle = g_A^i \; \bar{P}\gamma_\mu\gamma_5 P \; , \; i = u, d, s. \qquad (2)$$

The quantity of interest is the flavor singlet coupling

$$g_A^1 = \sum_{i=u,d,s} g_A^i. \qquad (3)$$

To determine g_A^1 on the lattice one needs to compute $Tr(\gamma_k\gamma_5 M^{-1})$ which implies off diagonal elements of M^{-1} in spin space[5]. Unfortunately the SET becomes inefficient for such matrix elements, as illustrated in figure 2. To improve the situation we resort to an explicit, component-wise treatment in spin space. This amounts to the application of one independent stochastic inversion per spin component (spin explicit method, SEM). We emphasize that SEM can be combined with PSQL.

The effect is illustrated in figure 3 and 4 for the standard error and the mean values on a single configuration. We observe earlier asymptotics in terms of the number of inversions (the factor 4 for SEM has been taken into account).

In order to test the sensitivity of the method for a determination of g_A^1 we have analysed 200

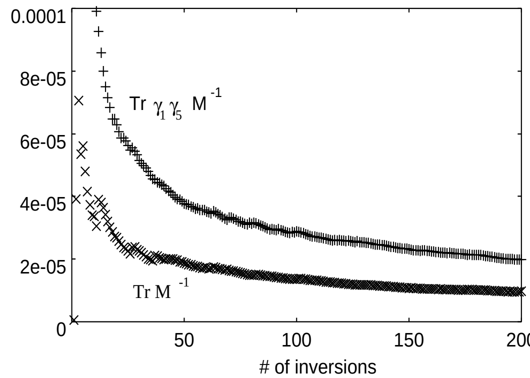

Figure 2. *The standard error $\sigma/\sqrt{N}$ on one configuration for $Tr(M^{-1})$ and $Tr(\gamma_1\gamma_5 M^{-1})$ with Z2 noise source vectors.*

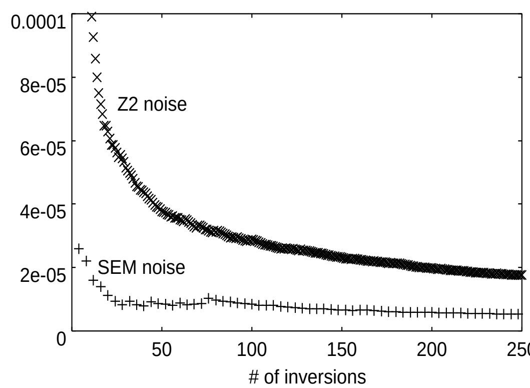

Figure 3. *The standard error $\sigma/\sqrt{N}$ on one configuration for $Tr(\gamma_1\gamma_5 M^{-1})$ for Z2 and SEM noise source vectors.*

SESAM configurations, at $\kappa = 0.157$. From the ratio

$$\begin{aligned} R(t)^{disc}_{g_A} \quad &= \quad \frac{\langle P_{11}(0 \to t) Tr(\gamma_3\gamma_5 M^{-1})\rangle}{\langle P_{11}(0 \to t)\rangle} \\ &\quad\; -\langle Tr(\gamma_3\gamma_5 M^{-1})\rangle \\ \xrightarrow{t\ large} \quad & \text{const} \quad +t\,\langle P_1|\bar{q}\gamma_3\gamma_5 q|P_1\rangle^{latt}_{disc} \qquad (4) \end{aligned}$$

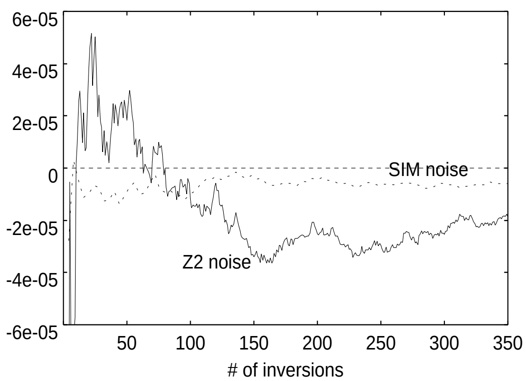

Figure 4. *Mean values for $Tr(\gamma_1\gamma_5 M^{-1})$ on one configuration for Z2 and SEM noise source vectors.*

we obtain signals as given in figure 5. Using both, SEM and PSQL, one finds the signal to stick clearly out of the noise!

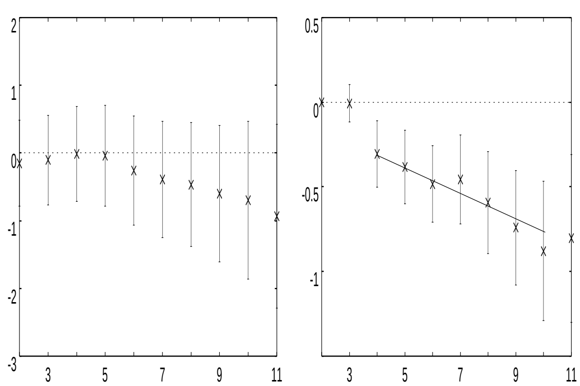

Figure 5. $R(t)^{disc}_{g_A}$ *measured without PSQL (left) and with PSQL on* 200 *SESAM configurations,* 100 *SEM noise source vectors,* $\kappa = 0.157$.

4. Conclusion and outlook

We have presented a variant of the SET which is suited to measure the flavor singlet axial vector coupling of the nucleon on a limited sample of QCD configurations. We can cope with the smallness of the axial vector signal by spin explicit inversions. Although this amounts to an increase in computational cost by a factor 4 for each estimate, the overhead turns out to be largely compensated by earlier asymptotics. The second ingredient of our proposal is the use of plateau sampling, which appears to reduce the fluctuations by a factor $2-3$, both for the scalar and for the axial insertions.

This encourages us to perform a full physics analysis to the complete set of SESAM configurations.

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